Developing Agent-Based Models of Business Relations and Networks*

(forthcoming bookchapter 2011)

Ian Wilkinson, Fabian Held
School of Marketing, Australian School of Business, University of New South Wales
Robert Marks
School of Economics, Australian School of Business, University of New South Wales
Louise Young
School of Marketing, College of Law and Business, University of Western Sydney
Sydney, NSW 2052, Australia
i.wilkinson@unsw.edu.au

Business relations and networks play a central role in the way business and economic systems are organised and function. But their dynamics and evolution have received limited research attention, with research focusing more on comparative static and cross-section surveys. In order to develop appropriate research-based management and policy advice we need a better understanding of how business relations and networks form and evolve. One way to do this is through the development of agent-based simulation models of business relations and networks that allow researchers to systematically explore the nature and impact of different factors on the structure, behaviour and performance that are beyond traditional closed-form mathematical solution and which would be impossible in the field. We identify the main mechanisms driving the evolution of business relations and networks and review models from various disciplines that attempt to represent these. This provides the basis for developing a generic simulation platform called the Business Network Agent-Based Modelling System (BNAS).

1.1 INTRODUCTION

Social and business networks are ubiquitous and an increasingly important area of research attention in many disciplines [18, 29, 63]. One of the major challenges is to better understand, predict and control their dynamics, including how they form and evolve and how this shapes their behaviour and performance [88, 95].

Business Networks refers to the interdependent systems of intra- and inter-organisational relations that are involved in markets, including firms, government agencies and other types of organizations [113]. They are increasingly understood as examples of complex adaptive systems (CAS) in which order arises in a bottom-up self-organising manner, based on the micro interactions taking place over time among the networks, people, and organisations involved [24, 72, 77, 113]. In addition, large scale or macro order has feedback effects on lower-level, micro interactions, and the network operates in the context of other connected networks and the broader environment. CAS theory challenges traditional notions of management, policy making and research methods because control is decentralised not centralised; CAS are highly non-linear systems exhibiting emergent behaviour and structure.

These characteristics of business networks are more clearly recognised and appreciated as economies become increasingly internationalised, interconnected and interdependent [57] as a result of revolutions in commerce and communication technologies that broaden, deepen and speed up the processes of interaction among economic actors around the world. Moreover, we recognise networks more clearly because network theories and research methods have developed rapidly over recent years, driven by developments in the science of networks and advances in computer technology.

The increasing attention being given to business networks is apparent in all business disciplines, where the nature, development and performance of firms, markets, regions and nations is increasingly being linked to the way economic actors are interconnected rather than being based solely on their individual characteristics. In marketing this focus can be traced to

* This chapter is based in part on papers presented in 2010 at the World Congress on Social Simulation (Kassel, Germany), Industrial Marketing and Purchasing (IMP) Group Annual Conference (Budapest, Hungary) and the Australia New Zealand Marketing Academy Annual Conference (Christchurch, New Zealand). The research is supported by an Australian Research Council Discovery Project Grant DP0881799 (2008-10).

Book Title
First Author & Second Author

research in channels and B2B markets in the US, Europe and Australasia [112] and is of continuing importance [49, 113, 115]. In management, strategy and economics the topic increasingly features in books, articles and special issues in leading management and economic journals [e.g. 29, 63, 84, 88].

The challenge for managers and policymakers is not about the management and control *of* such systems but how to participate and manage *in* them [90]. Government agencies and policy makers do not operate outside business networks: instead, they are part of them [107], along with the other people, firms and organisations. All participants are to some degree interdependent; it cannot be avoided. No one is in overall control, though some may have far more influence on parts of the system than others do. What happens depends on the ways different actors behave and respond and the direct and indirect interactions taking place over time among these actions and responses. Overall behaviour and performance is not a simple additive sum of the behaviour of the actors involved.

In order to provide research-based advice to managers and policy-makers, we need to understand the dynamics of business relations and networks, including how they form and evolve. Unfortunately, past research is dominated by linear, comparative-static, variable-based theories and methods, in which time and process are mostly absent. Structure, behaviour and context are summarised in terms of models of fixed entities (people, firms, relations, networks) with variable properties [1, 86] tested mainly via cross-sectional surveys of managers' perceptions and reports designed to measure the relevant variables and the patterns of co-variance resulting. A summary of the main variables and their posited links is shown in Figure 1.

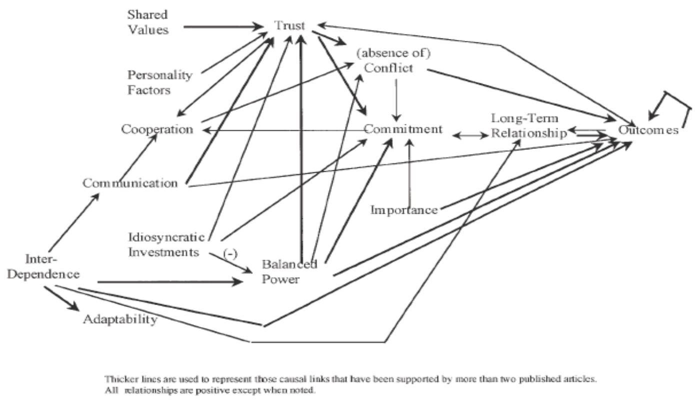

Figure 1 Links between dimensions of relations, source: [62]

Such research is of limited value in showing how real systems behave and evolve over time because they provide no understanding, tests or observations of the underlying causal mechanisms driving these. Variables do not exist and act in business systems, they are researcher abstractions produced by operational processes. Driving mechanisms may be alluded to in developing models and hypotheses but are not directly identified, modelled or tested.

There has been some previous research concerning the dynamics and evolution of business relations and networks [112]. This includes:

- Stage models of business relations, in which a pre-given sequence of stages is assumed to occur, with each providing the preconditions for the next, analysis of the processes involved in relation and network development [28, 40]
- Case studies and descriptive characterisations of relationship and network histories that highlight some of the processes going on [e.g. 6, 59, 66, 81, 91]
- Partial theories and schematic models suggesting some of mechanisms and feedback effects that drive the dynamics of aspects of relations, such as trust and power [e.g. 6, 59, 60, 81, 91, 111].
- Speculative descriptions characterising patterns of change and evolution [e.g. 50]
- Description and analysis of some mechanisms and processes involved in the dynamics and evolution of business relations and networks [e.g. 48, 52, 99, 111, 116]

- There are also more general theories of organisational change and development that are relevant to business relations and networks [e.g. 3, 104].

1.2 THE ROLE OF SIMULATION AND AGENT-BASED MODELS

One way of advancing research is through the use of computer simulations, especially agent-based methodologies, which are becoming ever more widely used and appreciated in science [45], including marketing [79, 106] and economics [101]. Simulation represents a third way of doing science in contrast to induction and deduction [8]. Its uses are manyfold, including: prediction, performing tasks, training, entertaining, educating, existence proofs and discovery [8]. For scientific research, the main goals are prediction, existence proofs and discovery.

One example of a successful discovery about the behaviour of networks achieved through simulation is [106], which challenges theories about the importance of critical individuals (opinion leaders) in diffusion processes. These simulations suggest that under most conditions large cascades of diffusion across social networks are driven not by influentials but by a critical mass of easily influenced individuals. In training and education, simulations have the potential to present abstract theories in a visually appealing and engaging way, e.g. a simulation of the "Beer Game" can give manager and business students a feel for the many interdependencies in supply chains [58], and agent-based models lie behind most of the online and computer games that have become popular, such as Second Life, Simlife, SimCity and The Sims. Along similar lines, "flight simulators" can be developed to allow managers and policy makers to experiment with alternative strategies in complex systems that are beyond analytical solutions, individual control and simple rules of thumb.

There has been some previous research using simulation methods to study business relations and networks. Starting in the 1960s, some computer simulations of business systems were developed, including Forrester's models of industrial dynamics [41], models of market processes [13] and logistics simulation models [19]. These models were limited by the lack of accessible computing power and suitable programming languages. They are all examples of System Dynamics models, in which the dynamics depends on starting conditions and the fixed structure imposed in the form of a set of differential equations linking variables related to representative actors. They do not allow structures to change over time or in response to behaviour, learning and outcomes.

More recently simulations have used Boolean rules to represent the way exchanges are connected [30, 32, 114] and to represent aspects of the evolution of a particular industry [39]. But no attempts have been made to develop more comprehensive simulation models that can be calibrated and tested against the dynamics and known histories of real relations and networks.

1.3 AGENT-BASED MODELS (ABM)

ABM represent a revolution in the way scientists can build models to replicate, analyse, test and predict the behaviour of CAS, because they are able to go beyond the gross simplifications and assumptions required to solve other types of models. Their relevance and importance is being increasingly recognised in many sciences, though less so in marketing and business, and this is particularly apparent in responses to the failure of traditional economic models to anticipate the global financial crisis, which is leading researchers, government and business to focus attention of building more realistic ABM of socio-economic systems [e.g. 14].

An ABM is a model formalized in computer code that represents a collection of formalized mathematical rules, applied to a clearly defined set of inputs [36, 45]. These rules transform the inputs into outputs, and, through systematic computation and analysis of the space of possible input values, a mapping of the respective outcomes can be calculated. In contrast to many mathematical models, simulation models need not be so strongly simplified that they become analytically tractable. ABM allow us to build, investigate and test models of complex systems that are beyond the reach of traditional analytical methods, though they may be summarised in algebraic form [73]. They are not restricted to general statistical models of behaviour, central driving equations or representative agents, but can represent in more realistic ways the micro interactions taking place and how they produce macro structures and patterns of behaviour. They are not based on a set of general driving equations, as in System Dynamics simulation methods, but allow for bottom-up control, via a heterogeneous set of interacting agents which could represent people and firms, as well as other actors and objects such as markets, environments and resources, each with their own characteristics, predispositions and rules of behaviour. These rules of behaviour can change as a result of learning via interaction and feedback effects [101]. Statistical and mathematical analysis and variable-based accounts of system behaviour are still relevant: they play an essential role in developing and testing ABM and in summarising their output, as they do for real-world social and economic systems.

ABM offer a middle ground between "thick" and "thin" descriptions [78]. Thick descriptions result from in-depth case studies of actual histories, which reveal the complex causal processes involved but cannot be easily generalised. Thin descriptions result from sample, survey-type research that is more generalisable but abstracts away from any examination of the processes, events or choices by which different types of variables are interrelated and affect outcomes. Between these two extremes, ABM simulations have been characterised as opaque thought experiments [25].

Instead of being limited to a study of what has happened, ABM allow us to test counterfactuals, to synthesise new forms of behaviour and organisation that have not existed in real life, which is why this approach is sometimes referred to as Artificial Life [2, 71]. "[Synthesis] extends the empirical database upon which the theory of the discipline is built beyond the often highly accidental set of entities that nature happened to leave around for us to study" [71 p. ix]. We cannot go back and rerun history to see how sensitive outcomes are to different factors and interventions, but ABM enable us to capture important features of the process, identify tipping points, and to conduct computer experiments of the impact of different factors [68]. Different theories or assumptions about a system can be implemented and examined.

1.4 EXPLAINING DYNAMICS: IDENTIFYING AND MODELLING CAUSAL MECHANISM

In order to develop realistic comprehensive models of the dynamics and evolution business networks that can incorporate and build on past descriptions, characterisations and partial theories, we need to focus on the underlying mechanisms and processes driving change, not on variables and their co-variance. Mechanisms are the verbs of social and business life, rather than the nouns and adjectives of variable-based research. The term "mechanism" is used loosely and can be confusing. It has to do with how something has an effect: the actual physical, mechanical, psychological, economic, sociological etc. processes that are involved in one thing leading to another: "a real process in a concrete system, such that it is capable of bringing about, or preventing, some change in the system as a whole" [21 p. 414]. More comprehensively: "Mechanisms consist of entities with their properties and the activities that these entities engage in, either by themselves or in concert with other entities...a constellation of entities and activities that are organized such that they regularly bring about a particular type of outcome." [53]. Managers and policy makers work with mechanisms and are players in various types of mechanisms driving business networks. They are actors in systems not variables and operate in terms of the sense-making processes and actions available to them and the consequences these have for themselves and others.

A focus on mechanisms and processes leads to a different approach to explanation that is more relevant for understanding dynamics and change. As summarised by Herbert Simon, "to 'explain' an empirical regularity is to discover a set of simple mechanisms that would produce the former in any system governed by the later" [7]. Mechanisms are often left implicit in our causal explanations, especially if we are focusing on the behaviour of variables rather than actors and events [22, 103]. Thus we may say that a manager's trust in another firm depends on how reliably or benevolently a firm has acted in the past. This implies that if the other firm's behaviour in terms of reliability and benevolence changes, then it could affect the amount of trust the manager has in it. But left unspecified are the processes involved: what kinds of changes in behaviour are perceived by the manager as evidence of better behaviour? How is this communicated to and understood by the manager? How likely are accurate perceptions to be due to thresholds of awareness and attention, bounded rationality and selective perception processes? How do changes in perception cause changes in trust? Is it a simple addition as suggested in some models of behaviour, or a more complex psychological sense-making process? What happens as a result of changes in the amount of trust, and how and why? Such questions cannot be answered by a variables based approach, though this remains relevant as a way of validating and testing ABM of mechanisms.

Mechanisms are everywhere because that is the way things work, but identifying them is not something social scientists are trained to do. Natural scientists and engineers are much more concerned with mechanisms as they try to understand how things work and use this knowledge to make other things. Instead of analysis, the focus is on synthesis. Analysis focuses on breaking up existing systems into parts and subsystems in order to understand how they work. Synthesis focuses attention on explaining by growing or replicating in a simulation some pattern of behaviour or outcomes; what Epstein (2006) refers to as generative social science [35].

Synthesis is the basis for major advances in science. This is reflected in developments in chemistry in the 1850s with the discovery of the table of the elements and how to make elements that did not exist in nature and, a century later in the revolution that took place in biology after the discovery of the structure of DNA. Social and economic systems are more complex because people have expectations about the future and the rules of behaviour are not fixed but change and evolve. Also, these systems are less amenable to systematic experimentation. But developments in computing power and ABM have opened up a new way to proceed. Similar revolutionary advances can be expected in business and social science when we are able to build ABM of the many psychological, social and economic mechanisms demonstrating how they interact over time to produce the business and social systems we observe. We can then use the same models to explore what could happen – economic and business life as it could be – or what is sometimes referred to as Artificial Life [71].

1.5 MECHANISMS DRIVING BUSINESS RELATIONS AND NETWORKS

A comprehensive cross-disciplinary review of previous research regarding business relations and networks resulted in the identification of five broad types of mechanisms and processes, as well as numerous examples of each type. These are summarised in columns 1 and 2 of Table 1. In addition we reviewed previous research in which one or more of the mechanisms identified was modelled in a precise way in the form of computer code, mathematical formulae or other precise specification. The review covered disciplines including biology, ecology, physics, engineering, artificial intelligence and the social, economic and business sciences [54, 55]. Examples were found for most but not all mechanisms and are illustrated in column 3 of Table 1.

Table 1 Business Relations and Network Mechanisms and Models

	Example mechanisms	**Example Models**
1. Business Acting and Specialising	Producing, consuming, buying, selling, learning, copying, targeting, in-sourcing, outsourcing, innovating, firm creation and demise	Economic models of Scale and scope efficiency, transaction cost and production functions [26] Choice and evaluation models [80] Imitation and learning models [20] Specialisation [33, 93] Central place and gravity models [4, 23])
2. Business Mating	Finding, being found, attracting, homophily, repelling, choosing, being chosen	Trade and Network formation models [16, 65, 106], Attraction and Gravity models [4, 23, 61, 94] Matching, alliance models [43] Partner and trade choice models [12, 74, 98, 109]
3. Business Dancing	Interacting, exchanging, cooperating, defecting, responding, initiating, trusting, liking, committing, learning, adapting, terminating	Iterated game models [9, 100, 117] Bargaining models [65, 67] Trust models [64, 102] Evolution of cooperation models [56, 70, 85] Learning models [20] Attraction and loyalty models [67]
4. Inter - Connecting	Enabling and constraining effects of other relations, comparing, accessing, prioritising	Learning models [20, 97] Competition and trade models [10, 97] Diffusion models [46, 96] Recommendation engines (e.g. Amazon) Network Co-Evolution models [34, 42, 47]
5. Environmental impact	History, enabling and constraining effects of exogenous environment	Starting conditions Parameter settings Location mapping Environment processes [11]

Acting and Specialising

The work of business comprises many types of interrelated activities that are divided up among those involved, and markets and other forms of exchange are used to access specialists' resources and outputs. This forms the basis of all economic and business systems. Firms specialise in particular assortments of activities they perform, which depend on their capabilities and knowledge and the decisions they make, such as about what to outsource or not and the profitability of offering to other firms. Outsourcing decisions involve comparing and evaluating alternative exchange possibilities, which can change as a result of variations in offers and capabilities and the mutual learning taking place. For example, scale and scope, learning and innovation affect costs and performance, which alter outsourcing opportunities and decisions. Opportunities arise for new types of firms, including specialist intermediaries and suppliers, leading to the creation and adaption of firms. Firms also go out of business and employees and resources move elsewhere [26, 111].

The actions of firms in producing and consuming things has been modelled in terms of various input-output or production functions. Models of the mechanisms of specialisation are not common, although economic theories of exchange, scale and scope efficiencies, and transaction-cost theory, point to the existence of various mechanisms related to specialisation within and between firms that can be modelled in various ways. Some ABM of the evolution of specialists exist, including Banknet, in which intermediaries arise from among the initial actors based on trading patterns, scale and scope efficiencies and transaction costs [93], and a dynamic model of role differentiation in social networks [33]. Experiments have also been carried out to simulate the evolution of trade and specialisation that suggest ways of modelling the various mechanisms involved [65]. Other general mechanisms such as learning, choosing and imitating have received far more modelling attention.

Mating

Potential trading partners encounter each other and choose, get chosen, accept or refuse to do business together and form relations. This could vary from random processes to ones that are influenced by past interactions, predispositions and communication networks. Changing or keeping partners depends on the evaluation, choice and learning processes of each party.

There are many examples of partner search and mating mechanisms in simulations, as it is one of the core mechanisms necessary to design a network simulation. Simple models of network generation use random pairing [37] or preferential attachment relative to the number of existing links [15]. Preferential attachment has been extended in many ways. Agents might lose attractiveness with age [27], or form relationships relative to activity [38], or performance [89] or based on prior successful cooperation [44]. Relationships can also be formed depending on expectations [100], or by copying links of established members [105]. Iterated games with choice and refusal of partners have been used to model the development of trading relations in which actors make and accept offers to trade based on their previous experience with a partner [72, 97]. Attraction and gravity models mostly developed in geography are also relevant [4, 23], as is Schelling's (1971) classic model of the processes of segregation [94].

Dancing

Relations begin with initial interactions and exchange mechanisms. The experience and outcomes of business and social interactions and exchanges over time lead to learning and to changes in resources, attraction, perceptions, beliefs and evaluations for all those involved. Repeated interactions produce, reproduce and change a relationship atmosphere of some kind [113] that involves various types of actor bonds developing or not, e.g. trust, commitment, power/dependence, cooperativeness, opportunism, empathy, understanding. These bonds affect willingness to engage in further interactions and exchange and the strategies used. Innovation also takes place in relations Relationship termination can occur due to endogenous of exogenous changes, including interactions with third parties and the general environment.

Modelling the way firms interact over time has been dealt with mainly in terms of various types of learning models and iterated games. Learning has been modelled in a number of ways to reflect the ways people (and hence firms) adapt their perceptions of each other and how this affects their behaviour. These include changes in trust, power and cooperation [64, 102]. Axelrod and Hamilton's (1981)[9] early simulations focused on the iterated prisoner's dilemma and the conditions under which cooperative strategies emerge. Later work has considered other types of games and the evolution of strategies as a result of the experience of and performance in interactions over time [76, 100]. Similar models have been used to investigate the evolution of cooperation and the evolution of group and network structures under conditions of individual and group selection [56, 70, 85]. In a simulation of the Marseilles fish market, buying and selling agents learn to engage in and reward loyal behaviour. Sellers can reduce their risk caused by demand volatility and buyers benefit from better service and discounts [67].

Inter-Connecting Relations

Interactions and atmosphere in one relation can affect, positively or negatively, interactions in others that are connected to it [e.g. 5, 17, 108]. This is due to the operation of various kinds of mechanisms, including the way exchanges are interconnected positively along value chains or competitively [30-32]. In addition, communication and diffusion processes spread information and ideas through networks, leading to learning and adaption.

Many simulations depict the way relations are interconnected in terms of the effects of network structure on activities performed, including impacts on learning strategies and the profits of traders [69]. Cooperative behaviour in strategic games is facilitated if networks are sparsely connected [82]. Preferential attachment tends to connect hubs with each other and it has been found that cooperation can spread from such a cluster to the entire network [92]. Network positions can affect individual payoffs [110] and the establishment of trust and reliability in one relationship can impede the success of another [64]. Networks for economic interactions and those for the exchange of information do not necessarily coincide. Diffusion models on networks also model the way information flows across networks [46]. Lastly, choice models necessarily involve linking relations when agents compare and evaluate alternatives partners.

Environmental impacts

Business relations and networks operate in a context of a more general environment that both enables and constrains what happens. The initial conditions of any model are part of the historical environment as they reflect the experience and outcomes of prior events that are not directly modelled. The environment can also be included in terms of patterns of change over time in key exogeneous conditions, including the expansion and contraction of markets and resources, technological and infrastructure changes, and business laws and customs. This approach is exemplified in the modelling of unfolding history of the Anasazi tribe over hundreds of years, in which the modellers included known weather and geological patterns as part of the environment [11]). Testing the implications of taxes and subsidies shows that they can facilitate the establishment of cooperative behaviour, which proves to be persistent, even after these are terminated [75]. More technical results show that the network structure is influenced by the payoffs of strategic games as well as the reliability of communication transmission [87]. Moreover, the relation between the time scales of actions on the network and the speed of the rewiring process of the network has been found to be important. A high rewiring speed of connections can essentially change the payoff structure of strategic games so that they favour cooperation [83] In addition, the environment can be incorporated as a its own model system that is coupled with the model of the focal business networks, as for example when a fish industry network is linked to an ecological model of fish stocks and fish behaviour, or to a broader economic model. The environment model responds to outcomes in the network model and generates events with which the network model has to cope.

Discussion

There are some areas where additional modelling effort is required to represent particular mechanisms. Only a few examples could be found for the development of specialization in simulations. While simulations can be initiated in terms of actors with different characteristics, roles and degrees of specialisation, modelling the processes by which these differences emerge and change over time is not so well developed.

There are many examples of models of partner search and mating mechanisms because it is a core mechanism in network formation. Many simulations deal with search mechanisms and their impact on network topologies that mimic empirical processes. Nevertheless, there seems to be no simulations allowing agents to negotiate the terms of a relationship.

Business dancing, the way firms interact, learn and adapt relations over time has been dealt with in terms of various types of learning models and iterated games. Simulations do not yet capture the richness and multidimensional nature of the evolution of relationships. In most simulations a connection either exists or not. Only [67] can be seen as an exception here.

Various mechanisms exist that capture the ways the behaviour and outcomes in one relation are connected to that in other relations in a network, including general structural and network position effects, diffusion processes and choice processes, rather than in terms of the types of connections identified in marketing theory (e.g. [5]).

Last, environmental impacts in terms of the heterogeneity or location of actors have not yet been included in simulations. Including such heterogeneity would allow a closer matching of simulations with actual business networks but necessarily restrict the generality of simulations. The environment can be controlled in various ways in terms of starting conditions, parameter values and imposed historical patterns of change in exogenous conditions and can be the subject of its own model, which is then coupled with the business network model.

1.6 BUSINESS NETWORKS AGENT-BASED MODELLING SYSTEM (BNAS)

In order to design, develop, calibrate and evaluate ABM of business relations and networks, expert programming skills are required. This is so even though there have been major advances in the development of more user-friendly, general purpose ABM systems such as SWARM REPAST, MASON and NetLogo. All require knowledge and expertise in objectoriented programming languages such as Java, C++ or NetLogo. This represents a major barrier to research in business, economics and marketing, including business relations and networks, because academics and research students generally have little if any training in programming.

Business, economic and social researchers are trained for the most part in linear, variable-based, comparative-static theory and methods rather than non-linear, mechanism and process-based, dynamic and evolutionary theory and methods. The emphasis is on mathematics and statistics, which, while relevant to designing, analysing and evaluating ABM, are not sufficient to build them. This situation may change in the future as future generations of academics and researchers grow up with computers and models of this type and more courses are offered to train them in ABM. Of course, researchers in business and economics can team up with expert programmers, but this limits their ability to become involved in the design and development of the ABM, and expert programmers have limited domain knowledge and insights to guide them.

To overcome this problem, we are engaged in developing an ABM system that can be used by researchers interested in studying business relations and networks that does not require programming expertise. We call this the Business Networks Agent-Based Modelling System (BNAPS). This will provide researchers, as well as educators, managers and policy makers, with a user-friendly modular, menu-driven, modelling environment that will enable them to develop ABM relevant to their own interests, in which they can monitor, summarise and analyse the results to enable sharing, publication and contribute to the further development of BNAS. It would be similar in some ways to the design of modelling systems being developed to examine the behaviour of power markets [98]. BNAS will be embedded in a general ABM platform such as RePast or MASON. Researchers must understand the types of business relations and networks and theories they want to model and test, including the relevant mechanisms and rules governing the behaviour of the economic and passive actors involved. The BNAS will be able to guide researchers in translating their theories and stylised descriptions of business relations and networks and their contexts into a functioning ABM in which they will be able to select among alternative ways of implementing various mechanism and rules for inclusion in the model, as well as being able to contribute new rules and mechanisms, and to be able to combine and link them into an integrated ABM model ready for analysis and testing.

The potential uses and value of BNAS include:

- Build models to reproduce known structural, dynamic and evolutionary patterns and to systematically examine their stability, behaviour and performance under counterfactual conditions that would be impossible to do in the field.
- Develop ABM to illustrate and test alternative theories of the dynamics and evolution of business relations and networks.
- Examine how different mechanisms and processes interact and perform under varying conditions.
- Examine and compare results using alternative implementations of key mechanisms and features of actual or proposed business networks;
- Examine and test the impact of different controllable (firm and policy) and non-controllable factors on network dynamics, development and performance.
- Develop "flight simulators" for managers, industries and policy makers in order to sensitise them to the often counterintuitive outcomes of complex non-linear systems and to explore the effects of different scenarios and types of environments, similar to uses in the military.

1.7 SUMMARY AND CONCLUSIONS

We have highlighted the nature, role and importance of business relations and networks to the functioning of business and the economy. We have also argued that research regarding the dynamics, formation and evolution of business relations and networks is underdeveloped yet essential if we are to develop scientifically sound advice and guidance for managers and policymakers. To make progress we argue for the greater use of ABM methods to simulate complex adaptive systems such as business relations and networks and we show how this can be done in terms of identifying and modelling key causal mechanisms and processes.

We have reviewed the literature in business, economics and marketing to identify key mechanisms affecting the dynamics and evolution of business networks. These have been grouped logically into five broad types and we identified previous attempts to codify examples of these types of mechanisms. While examples for each basic type are available, the congruence with empirically identified mechanisms is limited and requires further model development in order to represent them.

A barrier to the development of such ABM research is the limited training and expertise business, economic and social researchers have in relevant programming and ABM methods. To overcome this, we propose the development of a general purpose ABM platform that does not require sophisticated programming expertise, yet allows researchers to become involved in the detailed design, construction, analysis and testing of their models. We believe a similar approach could be used to advance ABM in other domains because issues of dynamics, development, evolution and complexity are fast becoming a major area of attention in many research domains and in the world at large.

The growing relevance and interest these issues is indicated most clearly with the award of the Nobel Prize in Economics to Elinor Ostrom in 2009 for her work complex systems and the management of common pool resources, to Thomas Schelling in 2005 for his work on the unintended macro outcomes of micro behaviour in complex systems, and to Vernon Smith in 2002 on the role of social relations and the self-organising properties of market systems (not to mention the earlier award to Herbert Simon). The relevance and importance of the types of fine-grained nuanced models of complex systems that ABM can provide is further indicated by the failure of traditional economic models to predict the recent global financial crisis. Existing models make heroic assumptions about the nature and rationality of the behaviour of the representative actors in their models, which have increasingly been increasingly highlighted, leading to calls for more realistic models [e.g. 14, 51].

1.8 REFERENCES

1. Abbott, A. (2001) *Time Matters: On Theory and Method* (University of Chicago Press, Chicago).
2. Adami, C. (1998) *Introduction to Artificial Life* (Springer, New York).
3. Aldrich, H. (1999) *Organizations Evolving* (Sage Publications, London).
4. Allen, P. M. and Sanglier, M. (1979). A Dynamic Model of Growth in a Central Place System, *Geogr. Anal.*, **11** (3), pp. 256-272.
5. Anderson, J. C., Håkansson, H., and Johanson, J. (1994). Dyadic Business Relationships within a Business Network Context, *J. Marketing*, **58** (4), pp. 1-15.
6. Ariño, A. and Torre, J. D. L. (1998). Learning from Failure: Towards an Evolutionary Model of Collaborative Ventures, *Organ. Sci.*, **9** (3), pp. 306-325.
7. Augier, M. and March, J. G. (2004) *Models of a Man: Essays in Memory of Herbert A. Simon* (MIT Press, Cambridge, MA).
8. Axelrod, R. (2006) *Handbook on Research on Nature-Inspired Computing for Economics and Management*, ed. Rennard, J.-P., Chapter 7 "Advancing the Art of Simulation in the Social Sciences," (Idea Group, Hershey PA) pp. 90-100.
9. Axelrod, R. and Hamilton, W. D. (1981). The Evolution of Cooperation, *Science*, **211** (4489), pp. 1390-1396.
10. Axtell, R. (2005). The Complexity of Exchange, *Econ. J.*, **115** (504), pp. F193-F210.
11. Axtell, R., Epstein, J. M., Deand, J. S., Gumermane, G. J., Swedlundg, A. C., Harburgera, J., Chakravartya, S., Hammonda, R., Parker, J., and Parkera, M. (2002). Population Growth and Collapse in a Multiagent Model of the Kayenta Anasazi in Long House Valley, *Proc. Natl. Acad. Sci. USA*, **99** (supp. 3), pp. 7275-7279.
12. Axtell, R. L., Robert, A., Epstein, J. M., and Cohen, M. D. (1996). Aligning Simulation Models: A Case Study and Results, *Comput. Math. Organ. Th.*, **1** (2), pp. 123-141.
13. Balderston, F. E. and Hoggatt, A. C. (1962) *Simulation of Market Processes*. IBER Special Publications (Institute of Business and Economic Research, Berkeley).
14. Ball, P. (2010). Model Citizens: Building Simearth, *New Sci.*, **2784**
15. Barabási, A.-L. and Albert, R. (1999). Emergence of Scaling in Random Networks, *Science*, **286** (5439), pp. 509-512.
16. Bianconi, G. and Barabási, A. L. (2001). Competition and Multiscaling in Evolving Networks, *Europhys. Lett.*, **54** (4), pp. 436-442.
17. Blankenburg-Holm, D., Eriksson, K., and Johanson, J. (1996). Business Networks and Cooperation in International Business Relationships, *J. Int. Bus. Stud.*, **27** (5), pp. 1033-1053.
18. Borgatti, S. P., Mehra, A., Brass, D. J., and Labianca, G. (2009). Network Analysis in the Social Sciences, *Science*, **323**, pp. 892-895.
19. Bowersox, D. J. (1972) *Dynamic Simulation of Physical Distribution Systems*. Msu Business Studies (Division of Research, Graduate School of Business Administration, Michigan State University, East Lansing).
20. Brenner, T. (2006) *Handbook of Computational Economics* **2**, eds. Tesfatsion, L. and Judd, K. L., Chapter 18 "Agent Learning Representation: Advice on Modelling Economic Learning," (Elsevier, Amsterdam) pp. 895-947.
21. Bunge, M. (1997). Mechanism and Explanation, *Philosophy of the Social Sciences*, **27** (4), pp. 410-

465.
22. Buttriss, G. and Wilkinson, I. F. (2006). Using Narrative Sequence Methods to Advance International Entrepreneurship Theory, *J. Int. Entrepreneurship*, **4**, pp. 157-174.
23. Clarke, M. and Wilson, A. G. (1983). The Dynamics of Urban Spacial Structure: Progress and Problems, *J. Reg. Sci.*, **23** (1), pp. 1-18.
24. Dagnino, G. B., Levanti, G., and Destri, A. M. L. (2008). Evolutionary Dynamics of Inter-Firm Networks: A Complex Systems Perspective, *Adv. Strategic Manage.*, **25**, pp. 67-129.
25. Di Paolo, E. A., Noble, J., and Bullock, S. (2000). Simulation Models as Opaque Thought Experiments, *Seventh International Conference on Artificial Life*. MIT Press, Cambridge, MA., pp. 497-506.
26. Dixon, D. F. and Wilkinson, I. F. (1986). Toward a Theory of Channel Structure, *Res. Market.*, **8**, pp. 27-70.
27. Dorogovtsev, S. and Mendes, J. (2000). Evolution of Networks with Aging of Sites, *Phys. Rev. E*, **62** (2), pp. 1842-1845.
28. Dwyer, F. R., Schurr, P., and Oh, S. (1987). Developing Buyer-Seller Relationships, *J. Marketing*, **51**, pp. 11-27.
29. Easley, D. and Kleinberg, J. (2010) *Networks, Crowds, and Markets: Reasoning About a Highly Connected World* (Cambridge University Press, Cambridge, UK).
30. Easton, G., Brooks, R. J., Georgieva, K., and Wilkinson, I. F. (2008). Understanding the Dynamics of Industrial Networks Using Kauffman Boolean Networks, *Adv. Complex Sys.*, **11** (1), pp. 139-164.
31. Easton, G., Wiley, J., and Wilkinson, I. F. (1999), Simulating Industrial Relationships with Evolutionary Models, *28th European Marketing Academy Annual Conference* (Humboldt University, Berlin).
32. Easton, G., Wilkinson, I. F., and Georgieva, C. (1997) *Relationships and Networks in International Markets*, eds. Gemünden, H. G., Ritter, T., and Walter, A., Chapter 17 "Towards Evolutionary Models of Industrial Networks - a Research Programme," (Elsevier, Oxford) pp. 273-293.
33. Eguíluz, V. M., Zimmermann, M. G., Cela-Conde, C. J., and Miguel, M. S. (2005). Cooperation and the Emergence of Role Differentiation in the Dynamics of Social Networks, *Am. J. Sociol.*, **110** (4), pp. 977-1008.
34. Ehrhardt, G. C. M. A., Marsili, M., and Vega-Redondo, F. (2006). Phenomenological Models of Socioeconomic Network Dynamics, *Phys. Rev. E*, **74** (3), pp. 036106-11.
35. Epstein, J. M. (2006) *Generative Social Science: Studies in Agent-Based Computational Modeling* (Princeton University Press, Princeton).
36. Epstein, J. M. (2008). Why Model?, *J. Art. Soc. Soc. Sim.*, **11** (4), pp. 12.
37. Erdös, P. and Renyi, A. (1959). On Random Graphs, *Publ. Math. Debrecen*, **6**, pp. 290-297.
38. Fan, Z. and Chen, G. (2004). Evolving Networks Driven by Node Dynamics, *Int. J. Mod. Phys. B*, **18** (17-19), pp. 2540-2546.
39. Følgesvold, A. and Prenkert, F. (2009). Magic Pelagic - an Agent-Based Simulation of 20 years of Emergent Value Accumulation in the North Atlantic Herring Exchange System, *Indus. Market. Manag.*, **38** (5), pp. 529-540.
40. Ford, D. (1980). The Development of Buyer-Seller Relationships in Industrial Markets, *Eur. J. Market.*, **15** (5/6), pp. 339-354.
41. Forrester, J. W. (1961) *Industrial Dynamics* (M.I.T. Press, Cambridge, MA).
42. Fronczak, P., Fronczak, A., and Holyst, J. A. (2006). Self-Organized Criticality and Coevolution of Network Structure and Dynamics, *Phys. Rev. E*, **73** (4), pp. 046117-4.
43. Gavrilets, S., Duenez-Guzman, E. A., and Vose, M. D. (2008). Dynamics of Alliance Formation and the Egalitarian Revolution, *PLoS ONE*, **3** (10), pp. e3293.
44. Gilbert, N., Pyka, A., and Ahrweiler, P. (2001). Innovation Networks - a Simulation Approach, *J. Art. Soc. Soc. Sim.*, **4** (3).
45. Gilbert, N. and Troitzsch, K. G. (2005) *Simulation for the Social Scientist*, 2 Ed. (Open University Press, Berkshire, UK).
46. Goldenberg, J., Libai, B., and Muller, E. (2001). Talk of the Network: A Complex Systems Look at the Underlying Process of Word-of-Mouth, *Market. Lett.*, **12** (3), pp. 211-223.
47. Gross, T. and Blasius, B. (2008). Adaptive Coevolutionary Networks: A Review, *J. Roy. Soc. Interface*, **5** (20), pp. 259-271.
48. Haase, M. and Kleinaltenkamp, M. (2011 in press). Property Rights Design and Market Process: Implications for Market Theory and S-D Logic, *J. Macromarket.*, **30**.
49. Håkansson, H., Harrison, D., and Waluszewski, A. eds. (2005) *Rethinking Marketing: Developing a New Understanding of Markets* (Wiley, San Francisco, CA).
50. Håkansson, H. and Johanson, J. (1993) *Industrial Networks* **5**, ed. Sharma, D. D., Chapter "Industrial Functions of Business Relationships," (JAI Press, New York) pp. 13-29.
51. Haldane, A. G. and May, R. M. (2011). Systemic Risk in Banking Ecosystems, *Nature*, **469** (7330), pp. 351-355.
52. Halinen, A. and Törnroos, J.-Å. (1998). The Role of Embeddedness in the Evolution of Business Networks, *Scand. J. Manage.*, **14** (3), pp. 187-205.

53. Hedström, P. (2005) *Dissecting the Social: On the Principles of Analytical Sociology* (Cambridge University Press, Cambridge, UK).
54. Held, F. (2010), Developing Agent-Based Models of Business Relations and Networks as Complex Adaptive Systems, *School of Marketing* (University of New South Wales, Sydney).
55. Held, F., Wilkinson, I. F., Marks, R., and Young, L. (2010), Exploring the Dynamics of Economic Networks: First Steps of a Research Project, *3rd World Congress on Social Simulation* (Kassel, Germany).
56. Henrich, J. (2004). Cultural Group Selection, Coevolutionary Processes and Large-Scale Cooperation, *J. Econ. Behav. Organ.*, **53** (1), pp. 3-35.
57. Hidalgo, C. A., Klinger, B., Barabasi, A.-L., and Hausmann, R. (2007). The Product Space Conditions the Development of Nations, *Science*, **317** (5837), pp. 482-487.
58. Holweg, M. and Bicheno, J. (2002). Supply Chain Simulation - a Tool for Education, Enhancement and Endeavour, *Int. J. Product. Econ.*, **78** (2), pp. 163-175.
59. Huang, Y. (2010), Understanding Dynamics of Trust in Business Relationships, *School of Marketing* (University of New South Wales, Sydney).
60. Huang, Y. and Wilkinson, I. (2006), Understanding Trust and Power in Business Relationships: A Dynamic Perspective, *International Marketing and Purchasing Group Conference* (Milan, Italy).
61. Huff, D. L. (1964). Defining and Estimating a Trading Area *J. Marketing*, pp. 373-378.
62. Iacobucci, D. and Hibbard, J. D. (1999). Toward an Encompassing Theory of Business Marketing Relationships (BMRS) and Interpersonal Commercial Relationships (ICRS): An Empirical Generalization, *J. Interact. Market.*, **13** (3), pp. 13-33.
63. Jackson, M. O. (2008) *Social and Economic Networks* (Princeton University Press, Princeton, NJ).
64. Kim, W.-S. (2009). Effects of a Trust Mechanism on Complex Adaptive Supply Networks: An Agent-Based Simulation Study, *J. Art. Soc. Soc. Sim.*, **12** (3), pp. 4.
65. Kimbrough, E. O., Smith, V. L., and Wilson, B. J. (2008). Historical Property Rights, Sociality, and the Emergence of Impersonal Exchange in Long-Distance Trade *Amer. Econ. Rev.*, **98** (3), pp. 1009-1039.
66. Kinch, N. (1993), The Long-Term Development of Supplier-Buyer Relation, *International Marketing and Purchasing Group Conference* (Bath, UK).
67. Kirman, A. P. and Vriend, N. J. (2001). Evolving Market Structure: An Ace Model of Price Dispersion and Loyalty, *J. Econ. Dynam. Control*, **25** (3/4), pp. 459-502.
68. Kleijnen, J. P. C. (2008) *Design and Analysis of Simulation Experiments*. International Series in Operations Research and Management Science (Springer, Berlin).
69. Ladley, D. and Bullock, S. (2008). The Strategic Exploitation of Limited Information and Opportunity in Networked Markets, *Computational Econ.*, **32** (3), pp. 295-315.
70. Ladley, D., Wilkinson, I. F., and Young, L. C. (2007), Group Selection and the Evolution of Cooperation, *9th European Conference on Artificial Life* (Lisbon, Portugal).
71. Langton, C. G. ed. (1996) *Artificial Life: An Overview* Complex Adaptive Systems. (MIT Press, Boston).
72. Lebaron, B. and Tesfatsion, L. (2008). Modeling Macroeconomies as Open-Ended Dynamic Systems of Interacting Agents, *Amer. Econ. Rev.*, **98** (2), pp. 246-50.
73. Leombruni, R. and Richiardi, M. (2005). Why Are Economists Sceptical About Agent-Based Simulations?, *Physica A*, **355** (1), pp. 103-109.
74. Li, S. X. and Rowley, T. J. (2002). Inertia and Evaluation Mechanisms in Interorganizational Partner Selection: Syndicate Formation among U.S. Investment Banks, *Acad. Manage. J.*, **45** (6), pp. 1104-1119.
75. Lugo, H. and Jiménez, R. (2006). Incentives to Cooperate in Network Formation, *Computational Econ.*, **28** (1), pp. 15-27.
76. Marks, R. E. (1989). Breeding Optimal Strategies: Optimal Behavior for Oligopolists, *Proceedings of the Third International Conference on Genetic Algorithms, George Mason University*. Morgan Kaufmann Publishers, pp. 198−207.
77. May, R. M., Sugihara, G., and Levin, S. A. (2008). Ecology for Bankers *Nature*, **451**, pp. 893-895.
78. Mckelvey, B. (2004). Towards a Complexity Science of Entrepreneurship, *J. Bus. Venturing*, **19** (3), pp. 313-341.
79. Midgley, D. F., Marks, R. E., and Cooper, L. G. (1997). Breeding Competitive Strategies, *Manage. Sci.*, **43** (3), pp. 257-275.
80. Mosekilde, E., Larsen, E. R., and Sterman, J. D. (1991) *Beyond Belief: Randomness, Prediction, and Explanation in Science*, eds. Casti, J. L. and Karlqvist, A., Chapter "Coping with Complexity: Deterministic Chaos in Human Decisionmaking Behavior," (CRC Press, Boston, MA) pp. 199-229.
81. Narayandas, D. and Rangan, V. K. (2004). Building and Sustaining Buyer-Seller Relationships in Mature Industrial Markets, *J. Marketing*, **68** (3), pp. 63-67.
82. Ohtsuki, H., Hauert, C., Lieberman, E., and Nowak, M. A. (2006). A Simple Rule for the Evolution of Cooperation on Graphs and Social Networks, *Nature*, **441**, pp. 502-505.
83. Pacheco, J. M., Traulsen, A., and Nowak, M. A. (2006). Coevolution of Strategy and Structure in Complex Networks with Dynamical Linking, *Phys. Rev. Lett.*, **97** (25), pp. 258103-4.

84. Parke, A., Wasserman, S., and A., R. D. (2006). New Frontiers in Network Theory Development, *Acad. Manage. Rev.*, **31** (3), pp. 560-568.
85. Pennisi, E. (2009). On the Origin of Cooperation, *Science*, **325** (5945), pp. 1196-1199.
86. Poole, M., Scott, A. H., Van De Ven, K. D., and Holmes, M. E. (2000) *Organizational Change and Innovation Processes: Theory and Methods for Research* (Oxford University Press, New York).
87. Pujol, J. M., Flache, A., Delgado, J., and Sangüesa, R. (2005). How Can Social Networks Ever Become Complex? Modelling the Emergence of Complex Networks from Local Social Exchanges, *J. Art. Soc. Soc. Sim.*, **8** (4), pp. 12.
88. Rauch, J. E. (2010). Does Network Theory Connect to the Rest of Us? A Review of Matthew O. Jackson's Social and Economic Networks, *J. Econ. Lit.*, **48** (4), pp. 980-986.
89. Ren, J., Wu, X., Wang, W. X., Chen, G., and Wang, B. H. (2006). Interplay between Evolutionary Game and Network Structure: The Coevolution of Social Net, Cooperation and Wealth, *Arxiv preprint physics/0605250v2*.
90. Ritter, T., Wilkinson, I. F., and Johnston, W. (2004). Firms' Ability to Manage in Business Networks: A Review of Concepts, *Indus. Market. Manag.*, **33** (3), pp. 175-183.
91. Rond, M. D. and Bouchikhi, H. (2004). On the Dialectics of Strategic Alliances, *Organ. Sci.*, **15** (1), pp. 56-69.
92. Santos, F. C. and Pacheco, J. M. (2005). Scale-Free Networks Provide a Unifying Framework for the Emergence of Cooperation, *Phys. Rev. Lett.*, **95** (9), pp. 98104.
93. Sapienza, M. D. (2000) *Economic Simulation in Swarm*, eds. Luna, F. and Stefansson, B., Chapter 6 "An Experimental Approach to the Study of Banking Intermediation: The Banknet Simulator," (Kluwer, Boston) pp. 159-179.
94. Schelling, T. (1971). Dynamic Models of Segregation, *J. Math. Soc.*, **1**, pp. 143-186.
95. Schweitzer, F., Fagiolo, G., Sornette, D., Vega-Redondo, F., Vespignani, A., and White, D. R. (2009). Economic Networks: The New Challenges, *Science*, **325** (5939), pp. 422-425.
96. Simoni, M., Tatarynowicz, A., and Vagnani, G. (2006). The Complex Dynamics of Innovation Diffusion and Social Structure: A Simulation Study, *WCSS 2006 - The First World Congress on Social Simulation, Kyoto, Japan*.
97. Stanley, E., Ashlock, D., and Smucker, M. (1995) *Advances in Artificial Life*, 4.4 "Iterated Prisoner's Dilemma with Choice and Refusal of Partners: Evolutionary Results," (Springer, Berlin) pp. 490-502.
98. Sun, J. and Tesfatsion, L. (2007). Dynamic Testing of Wholesale Power Market Designs: An Open-Source Agent- Based Framework, *Computational Econ.*, **30** (3), pp. 291-327.
99. Sydow, J., Schreyögg, G., and Koch, J. (2009). Organizational Path Dependence: Opening the Black Box, *Acad. Manage. Rev.*, **34** (4), pp. 689-709.
100. Tesfatsion, L. (1997). A Trade Network Game with Endogenous Partner Selection, *Comp. Approach. Econ. Prob.*, pp. 249-269.
101. Tesfatsion, L. and Judd, K. L. eds. (2006) *Handbook of Computational Economics* 2. Handbooks in Economics Vol. 13. (Elsevier, Amsterdam).
102. Tomassini, M., Pestelacci, E., and Luthi, L. (2010). Mutual Trust and Cooperation in the Evolutionary Hawks-Doves Game, *BioSystems*, **99** (1), pp. 50-59.
103. Van De Ven, A. H. and Engleman, R. M. (2004). Event and Outcome Ddriven Explanations of Entrepreneurship, *J. Bus. Venturing*, **3**, pp. 343-358.
104. Van De Ven, A. H. and Poole, M. S. (2005). Alternative Approaches for Studying Organizational Change, *Organ. Stud.*, **26** (9), pp. 1377-1404.
105. Vázquez, A. (2000). Knowing a Network by Walking on It: Emergence of Scaling, *arXiv:cond-mat/0006132v4*.
106. Watts, D. J. and Dodds, P. S. (2007). Influentials, Networks, and Public Opinion Formation, *J. Cons. Res.*, **34** (4), pp. 441-458.
107. Welch, C. and Wilkinson, I. F. (2004). The Political Embededness of International Business Networks, *Int. Market. Rev.*, **21** (2), pp. 216-231.
108. Wiley, J., Wilkinson, I. F., and Young, L. (2009). A Comparison of European and Chinese Supplier and Customer Functions and the Impact of Connected Relations, *J. Bus. Ind. Market.*, **24** (1), pp. 35-45.
109. Wilhite, A. (2001). Bilateral Trade and Small-World Networks, *Computational Econ.*, **18**, pp. 49-64.
110. Wilhite, A. (2006) *Handbook of Computational Economics* **2**, eds. Tesfatsion, L. and Judd, K. L., Chapter 20 "Economic Activity on Fixed Networks," (Elsevier, Amsterdam) pp. 1013-1045.
111. Wilkinson, I. F. (1990). Toward a Theory of Structural Change and Evolution in Marketing Channels, *J. Macromarket.*, **10**, pp. 18-46.
112. Wilkinson, I. F. (2001). A History of Network and Channels Thinking in Marketing in the 20th Century, *Australas. Market. J.*, **9** (2), pp. 23-52.
113. Wilkinson, I. F. (2008) *Business Relating Business - Managing Organisational Relations and Networks* (Edward Elgar, Cheltenham, UK).
114. Wilkinson, I. F., Wiley, J., and Easton, G. (1999), Simulating Industrial Relationships with Evolutionary Models, *28th European Marketing Academy Annual Conference* (Berlin, Germany).

115. Woodside, A. G. ed. (2010) *Organizational Culture, Business-to-Business Relationships, and Interfirm Networks* 16. Advances in Business Marketing & Purchasing (Emerald Books, Bingley, UK).
116. Young, L. and Wilkinson, I. F. (2004), Evolution of Networks and Cognitive Balance, *International Marketing and Purchasing Group Conference* (Copenhagen, Denmark).
117. Zimmermann, M. G., Eguíluz, V. M., and San Miguel, M. (2004). Coevolution of Dynamical States and Interactions in Dynamic Networks, *Phys. Rev. E*, **69** (6), pp. 065102.